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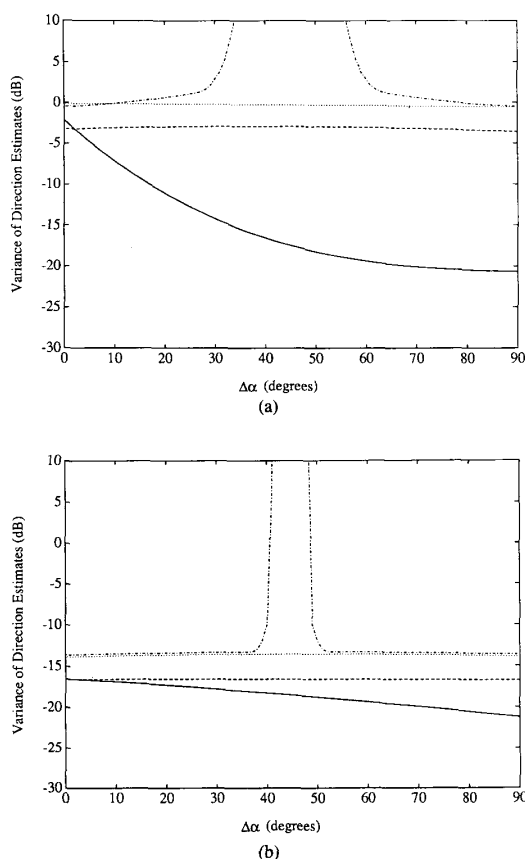


Fig. 5. Variance of $\hat{\theta}$ versus $\Delta\alpha$. Solid curve: full polarization method; dashed curve: angle-only method; dotted curve: x-axis dipoles only; dash-dot curve: y-axis dipoles only. (a) $\Delta\theta = 4^\circ$. (b) $\Delta\theta = 20^\circ$.

the difference in the signal directions is small, but the difference in the polarizations is large. Otherwise, the two approaches have similar performance.

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Fields from Point Sources Using the Aperture Field Method

Jørgen Appel-Hansen

Abstract—It is shown that the field of arbitrary point sources can be found from the aperture field method. It is exemplified that the exact result in the far field is easily obtained for an electric Hertzian dipole in free space.

I. INTRODUCTION

One method used in the calculation of the field radiated by aperture antennas is based on the Fourier transform. For the same antenna, Fourier transforms may be taken of various field and current distributions in the aperture plane. However, they give different results when the distributions are known only approximately. Usually, the apertures are also large in terms of the wavelength. In the present communication, the field distribution in the aperture plane is decomposed in distributions which correspond to those of point apertures in perfectly conducting electric or magnetic screens. Exact results are obtained in the far field.

The initial point source can be an arbitrary combination of electric and magnetic dipoles as well as loops. These can be decomposed in tangential dipoles and/or loops having a field component (electric field intensity \vec{E} or magnetic field intensity \vec{H} depending on the type of source) normal to the aperture plane. For each of these sources, the tangential fields, \vec{E}_{tan} or \vec{H}_{tan} , are zero except at the source point where the field is singular. The singularities can be specified by the delta function and its derivative. These are easily Fourier transformed and the total field of the initial source is found by superposition of the fields from each individual source.

The method is illustrated for an electric Hertzian dipole in free space. For further details the reader is referred to [1]. The purpose of the communication is to present cases for which the aperture field method gives exact results. Such cases do not seem to have been described in the general literature. However, it should be noted that in [2] a line aperture with a field corresponding to a magnetic line current is considered. While the expression derived in [2] is claimed only to be valid in front of the aperture, the expression derived here is valid in the whole space. The method is an alternative to the use of potential theory. Due to the employment of a general far-field expression, the method seems to provide the $\hat{\theta}$ - and $\hat{\phi}$ -components of the radiated field more easily.

Consider the electric Hertzian dipole at the origin O of an xyz -coordinate system and oriented in the z -direction as shown in Fig. 1. Let its current be given by

$$\vec{J}_e^{(d)} = \delta(x) \delta(y) \delta(z) I \hat{z} \quad (1)$$

where I is the dipole moment. In the following μ_0 , ϵ_0 , ω , and k are the permeability, permittivity, angular frequency and propagation constant, respectively, of free space. The time factor is $e^{j\omega t}$.

The equivalent tangential unit source is a loop with current $\vec{J}_m^{(l)}$ given by

$$\vec{J}_m^{(l)} = -\frac{I}{j\omega\epsilon_0} \delta(z) \{ \delta(x) \delta'(y) \hat{x} - \delta'(x) \delta(y) \hat{y} \}. \quad (2)$$

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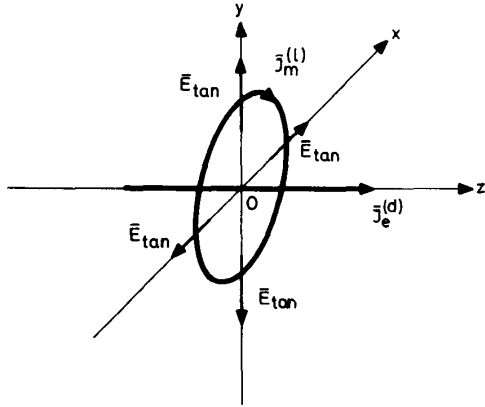


Fig. 1. Equivalent currents and aperture field.

By using field equivalence principles it is found that the far field may be determined from the aperture plane (xy -plane) distribution of the tangential electric field intensity \bar{E}_{tan} given by

$$\bar{E}_{\text{tan}} = -\frac{\Pi}{2j\omega\epsilon_0} \{ \delta'(x)\delta(y)\hat{x} + \delta(x)\delta'(y)\hat{y} \}. \quad (3)$$

This corresponds to a point aperture in a perfectly electrically conducting screen. According to [3], the radiation electric field intensity $\bar{E}(\bar{r})$ at \bar{r} in a usual spherical r, θ, ϕ -coordinate system is given by

$$\bar{E}(\bar{r}) = \frac{jk}{2\pi} \frac{e^{-jkr}}{r} \left[(f_x \cos \phi + f_y \sin \phi) \hat{\theta} + (-f_x \sin \phi + f_y \cos \phi) \cos \theta \hat{\phi} \right] \quad (4)$$

where

$$f_x \hat{x} + f_y \hat{y} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \bar{E}_{\text{tan}} e^{jk \sin \theta \cos \phi x + jk \sin \theta \sin \phi y} dx dy. \quad (5)$$

Thus

$$\begin{aligned} f_x &= -\frac{\Pi}{2j\omega\epsilon_0} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \delta'(x)\delta(y) e^{jk \sin \theta \cos \phi x + jk \sin \theta \sin \phi y} dx dy \\ &= \frac{\Pi k \sin \theta \cos \phi}{2\omega\epsilon_0} \end{aligned} \quad (6)$$

and similarly

$$f_y = \frac{\Pi k \sin \theta \sin \phi}{2\omega\epsilon_0}. \quad (7)$$

Insertion of (6) and (7) into (4) gives the well-known expression for the \bar{E} -field of the Hertzian dipole

$$\bar{E}(\bar{r}) = \frac{jk}{4\pi} \frac{e^{-jkr}}{r} \Pi \sqrt{\frac{\mu_0}{\epsilon_0}} \sin \theta \hat{\theta}. \quad (8)$$

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